(22–25). All other terms in this equation are the same as in Eq. (15) and $\hat{x}_n$ is given by Eq. (19).

Performance Characteristics

The performance characteristics of the empirical Bayes filter were investigated by Monte Carlo simulation for the case $p = r = 6$. The ratio of the trace of the averaged squared error matrix to the trace of the matrix S_n was used as a scalar measure of performance. Thus a decreasing performance ratio is associated with an increase in the performance of the filter.

In Ref. 5 it was found that the performance of an empirical Bayes estimator for estimating the mean of a multivariate normal distribution depended upon a summary quantity Z_j. If Q is the covariance matrix of u_n, the summary quantity here becomes

$$Z_j = [S_n - (Q^{-1} + S_n^{-1})^{-1}]_{jj}/[(Q^{-1} + S_n^{-1})^{-1}]_{jj} \quad (29)$$

An average summary quantity Z was defined as

$$Z = \sum_{j=1}^{r} Z_j/r \quad (30)$$

For a fixed n, it was found that the performance ratio just defined decreased as Z increased. Also, for a given situation, the performance ratio was observed to decrease monotonically with n until around $n = 25$ after which the performance ratio was essentially flat. These results agree with those of Ref. 5. It was also observed that the performance of this filter was fairly stable when components of u were generated from different shaped distributions except for a U-shaped distribution for which it was considerably better. This discrepancy is explained in Ref. 5.

Conclusions

An empirical Bayes filter has been developed for estimating the state of a discrete time linear system with linear observations. Some distributional assumptions on the state disturbance error as well as some distributional assumptions on the initial state vector have been relaxed. Knowledge of the forms of these distributions is not required for use of the filter. It is required that the state disturbance error be uncorrelated over time and be independent of the observation error and that the state error distribution remain stationary over time.

The performance of the filter has been examined by means of Monte Carlo simulation. It has been found that the performance of the filter does not depend significantly on the form of the state disturbance error distribution used in the simulation.

It has also been observed that the performance of the empirical Bayes filter depends on the relative magnitude of the observation error and state disturbance error covariance matrices, all other parameters being held fixed. An index has been presented which summarizes this relative magnitude, and the performance of the filter has been found to increase as this index increases.

References

[1] Weiss, I. M., "A Survey of Discrete Kalman-Bucy Filtering with Unknown Noise Covariances," AIAA Paper 70-955, Santa Barbara, Calif., 1970.

[2] Martz, H. F. and Born, G. H., "Empirical Bayes Estimation of Observation Error Variances in Linear Systems," AIAA Paper 70-1058, Santa Barbara, Calif., 1970.

[3] Robbins, H., "An Empirical Bayes Approach to Statistics," *Proceedings of the Third Berkeley Symposium on Probability and Statistics*, Vol. 1, 1955, pp. 157–163.

[4] Atchison, T. A. and Martz, H. F., eds., *Proceedings of the Symposium on Empirical Bayes Estimation and Computing in Statistics, Mathematics Series No. 6*, Texas Tech Press, Lubbock, Texas, 1970.

[5] Martz, H. F. and Krutchkoff, R. G., "Empirical Bayes Estimators in a Multiple Linear Regression Model," *Biometrika*, Vol. 56, No. 2, 1969, pp. 367–374.

[6] Wasan, M. T., *Parametric Estimation*, McGraw-Hill, New York, 1970.

[7] Kamat, S. J., *Application of Empirical Bayes Decision Procedures To Discrete Time Linear Filtering*, Ph.D. dissertation, 1970, Texas Tech Univ., Lubbock, Texas.

[8] Deutch, R., *Estimation Theory*, Prentice-Hall, Englewood Cliffs, N. J., 1965.

Separation of a Supersonic Accelerated Flow over Notches

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SEPARATED cavities in notches and grooves in the boundary of a uniform supersonic flow have been described extensively; the present Note discusses the effect of a negative pressure gradient (i.e., freestream acceleration) on the flow over such notches. Such situations occur in nozzle flows, near the nose of blunt hypersonic vehicles, etc.

Experiments were conducted in a wind tunnel fitted with nozzle blocks which generated a linear negative pressure gradient over the test section. Reference 1 gives detailed pressure distributions over the floor of the cavity and impact-pressure profiles through the shear layer. The equipment is described in Ref. 1 (also Ref. 2, which reports on similar studies of down-stream-facing steps).

Rectangular notches 0.5 and 1 in. deep with variable lengths were tested. The Mach number immediately ahead of separation was 1.81. The pressure gradient (and therefore the Mach number gradient) in the undisturbed stream over the region of the notch is characterized by a scale length

$$\lambda = -[(P_t/P_0)d(P/P_0)/dx]^{-1} \quad (1)$$

where P_0 is the static pressure immediately upstream of separation. The values of λ^{-1} used in these experiments was ∞ (uniform flow), 9.17 and 5.77 (in.). The upstream boundary layer was fully turbulent and the ratio of its thickness relative to the notch depth varied between 0.2 and 0.4.

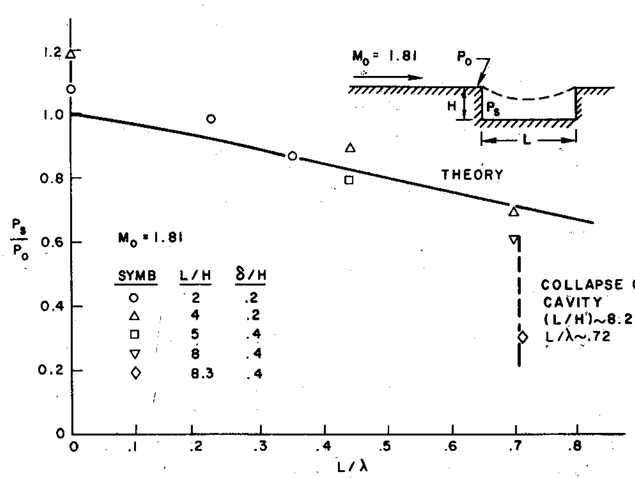

Fig. 1 Variation of the cavity pressure ratio in accelerated flow.

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When the pressures on the floor of the cavity are related to the pressure in the separation corner (Fig. 1), the distributions of the ratio are found to vary with cavity depth to length ratio as in uniform flow. However, the mean pressure level in the cavity depends on the degree of acceleration of the freestream and can be substantially lower than in uniform flow. The data are shown on Fig. 1. Figure 2 shows a schlieren picture of the shear layer over the notch. Indeed, the shear layer dips into the cavity upon separation with an attendant expansion.

If we assume that the perturbation of the freestream because of the cavity is small and the flow is isentropic, we can express the characteristic length λ^{-1} in terms of the freestream Mach number gradient

$$\lambda^{-1} = \gamma M_0\{1 + [(\gamma - 1)/2]M_0^2\}^{(1-\gamma)/(\gamma-1)}dM/dx \quad (2)$$

where M_0 is to be taken as the Mach number over the boundary immediately ahead of separation. The deflection of the separation streamline into the cavity at separation is given by

$$-\frac{d\theta}{dM} \cong \frac{1}{M_0}(M_0^2 - 1)^{1/2}\bigg/\left(1 + \frac{\gamma - 1}{2}M_0^2\right) \quad (3)$$

Combining, the shape of the separating streamline (origin of coordinates at the separation corner) is found

$$-y/\lambda = [(M_0^2 - 1)^{1/2}/2\gamma M_0^2](x/\lambda)^2 + \theta_s(x/\lambda) \quad (4)$$

where $(-\theta_s)$ is the sudden (Prandtl-Meyer) deflection into the cavity at the separation corner. Imposing a "closure condition" that the separation-streamline reattaches to the downstream edge of the notch allows a solution for θ_s and hence for the pressure inside the cavity (identified with the pressure in the separation corner). This is plotted on Fig. 1. The correlation with measurements is good.

The existence of an externally imposed pressure gradient must obviously also affect the critical length of the notch for which the flow can no longer "jump" across it entirely. Models which do not take into account the interaction between the "dead" air in the cavity and the external stream cannot deal with the closure problem. It is observed that even in the absence of an external pressure gradient the shear layer dips progressively further into the cavity as its length is increased. In a turbulent, reasonably thin, upstream boundary layer at intermediate supersonic Mach numbers, closure occurs at

$$(L/H)_{cr} \cong 11; \quad \lambda^{-1} = 0 \quad (5)$$

Fig. 2 Schlieren photograph of the shear layer over a notch: $L/H = 4$, $\lambda = 5.77$ in., $M_0 = 1.89$.

Assuming that the inward deflection of the shear layer due to this interaction and that due to the acceleration in the freestream are simply additive, we can estimate the critical closure length of rectangular notches from

$$\left(\frac{H}{L}\right)_{cr} = \left(\frac{H}{L}\right)_{cr,\lambda^{-1}=0} + \frac{d(y/L)_{max}}{d\lambda}\lambda = 0.091 + \frac{(M_0^2 - 1)^{1/2}}{8\gamma M_0^2}\left(\frac{L}{\lambda}\right)_{cr}$$

In these experiments the ½-in.-deep cavity collapsed at $(L/\lambda)_{cr} = 0.72$; $(L/H)_{cr} = 8.2$; $\lambda^{-1} = 0.173$ in.$^{-1}$. The previous equation yields a value of $(L/H)_{cr}$ of 8.3. It appears that the model is successful.

References

[1] Carichner, G. E., "Accelerated Supersonic Flow over Notches," M.S. thesis, June, 1968, Department of Engineering, University of California, Los Angeles, Calif.

[2] Charwat, A. F., Burghart, G. H., and Nunick, W. H., "Base Wakes in Accelerated Supersonic Free Streams," *Proceedings of the 1967 Heat Transfer and Fluid Mechanics Institute*, edited by P. A. Libby, D. B. Olfe, and W. V. Alla, Stanford University Press, 1967.

Mechanism of Entrainment in Turbulent Wakes

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THE existence of an abrupt limit to the region of turbulent flow is characteristic of wakes and free shear turbulence in general. The interface between the laminar and turbulent regions is a continuous but highly convoluted surface which is responsible for the intermittently turbulent output of a fixed probe placed in the transition region. Although considerable effort has gone into understanding the nature of the flow near this interface, it is still not clear how the turbulence spreads.

Townsend[1] has suggested that wake growth is essentially a surface phenomenon. In his view the basic mechanism of wake growth is a process of small scale nibbling by the turbulence at the interface. Like a flame front the wake thus advances at an essentially uniform rate over its entire surface. In addition the turbulence gives the wake a qualitative resemblance to an elastic solid so that initially random surface irregularities are unstable. Their growth leads to an increase of the total surface area and hence an increase of the entrainment rate.

However, it may be argued that Townsend's scheme does not go far enough. He used a simple model for the turbulent wake: an elastic jelly surrounded by an inviscid freestream. To simplify the computations it was also assumed that there was no mean velocity profile within the wake and consequently no straining of the turbulence. Mobbs[2] made an experimental investigation of just such an unstrained free turbulent flow. While he found that the boundary did indeed develop deep convolutions, he also observed that there was no entrainment of laminar fluid by this "wake." The apparent growth of the wake was due to the increasing amplitude of the

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